ANCOVA

http://www.unt.edu/rss/class/mike/6810/ANCOVA.pdf

What is Analysis of Covariance?

- When you think of ANCOVA, you should think of sequential regression, as it can be conducted as such
  - Covariate(s) enter in step 1, categorical predictor after
  - Want to assess how much variance is accounted for in the DV after controlling for (partialing out) the effects of one or more continuous IV-covariates

- ANCOVA always has at least 1 or more categorical, grouping IVs, and 1 or more continuous covariates).

- Covariate:
  - Want high correlation with DV; low with other covariates
  - Want few covariates
    - Recall that you are partialling out variance in the DV, having the group variable explain very little leftover variance is unappealing
  - If covariate correlates with IV → heterogeneity of regression
    - This is a violation of an assumption in ANCOVA, but a focus of interest in other approaches (Multilevel modeling)

Similarities with ANCOVA

- Like MR, it can be seen as an intermediate multivariate procedure with only one DV
- Extends ANOVA: Assess significant group differences on 1 DV
  - While controlling for the effect of one or more covariate
- Can be seen as Multiple Regression with 1+ continuous predictors (covariates) and 1+ dummy coded factors
- Allows for greater sensitivity than with ANOVA under most conditions

When to Use ANCOVA?

- One will see it used in two primary ways, one good, the other invalid by most accounts.
- Good: Experimental design
  - Manipulation of IV
  - Random Selection of Subjects
  - Random Assignment to Groups
  - IV does not affect the Covariate!![1]
- There are variables that might relate to the DV but one wants to control for them, i.e. partition out their variance from the residual variance
  - Leads to more statistical power, though the raw effect size should not change
    - Adjusted mean difference is the same as the adjustment is equal for the groups involved, and if one follows Kline and others' suggestion, the standardized effect *d* would not change.
- You may also see ANCOVA used as a followup procedure in MANOVA (again assuming experimental design)

When *not* to Use ANCOVA?

- The other way one might see an ANCOVA used is with intact groups[1]
- The idea is to 'equate' the groups on the covariate and see if any differences remain on the grouping variable
- Problems:
  - Makes pretty much no sense to equate groups that are not equal
  - The wrong model is probably being tested
  - You are partialling out some amount variance the factor would share with the DV
  - You will likely violate the Homogeneity of Regression assumption
- In general, one should ask 'Am I doing an experiment?'[2]. If the answer is no, use some other statistical procedure.
  - Regular MR
  - Multilevel modeling

ANCOVA Background Themes

- *Measures*: Group IV, Continuous Covariate, Continuous DV
- *Methods*: Inferential with experimental design & assumptions
- *Assumptions*:
  - Normality
  - Linearity
  - Homoscedasticity
  - Homogeneity of Regression:
    - Slopes between covariate and DV are similar across groups
    - Indicates no interaction between IV and covariate
    - If slopes differ, covariate behaves differently depending on which group (i.e., heterogeneity of regression)
    - When slopes are similar (what we desire for ANCOVA), Y is adjusted similarly across groups

Testing for Homogeneity of Regression

- First run the ANCOVA model with a Treatment X Covariate interaction term included[1]
- If the interaction is significant, assumption violated
  - Depending on the level of treatment, the relationship b/t covariate and DV changes
- If not, rerun without interaction term

- Or simply run the Covariate-DV regression for each group and assess in that way.[2]

ANCOVA in Terms of the General Linear Model

- $Y = GM_y + \tau + [B_i(C_i - M_{ij}) + \ldots] + E$
  - Y is a continuous DV
  - GM_y is grand mean of DV
  - τ is treatment effect
  - B_i is regression coefficient for *ith* covariate, C_i
  - M is the mean of ith covariate
  - E is error
- ANCOVA is an ANOVA on Y scores in which the relationships between the covariates and the DV are partialed out of the DV.
  - $Y - B_i(C_i - M_{ij}) = GM_y + \tau + E$

Central Themes for ANCOVA

- *Variance*: in DV
  - The linear combination that is examined in ANCOVA is the Y score that is adjusted for the effects of the covariates
- *Covariance*: between DV & Covariate(s)
  - in ANCOVA we can examine the proportion of shared variance (i.e., $\eta 2$) between the adjusted Y score and the IV(s).
- *Ratio*: Between Groups/ Within Groups
  - Just as with ANOVA, in ANCOVA we are very interested in the ratio of between-groups variance over within-groups variance.

ANCOVA Macro-Assessment

- *Effect Size:* ES= η^2
  - $\eta^2 = SS_{EFFECT} / SS_{TOTAL}$, after adjusting for covariates

- *F-test*
  - The statistical test in ANCOVA utilizes the statistic as in ANOVA
  - If significant, group means statistically differ after controlling for the effect of 1+ covariates

ANCOVA Micro-Assessment

- *Test of Means*, Group Comparisons
  - *Follow-up planned comparisons (e.g., FDR)*
- d-family effect size
  - While one might use adjusted means, if using experimental design the difference should be pretty much the same as original means. And typically you'll probably want to go with the regular means
  - However, current thinking is that the standardizer should come from the original metric, so run just the regular ANOVA and use the sqrt of the mean square error from that analysis

$$d = \frac{M_1 - M_2}{\sqrt{MS_{error}}}$$

- *Graphs of (adjusted) means* for each group also provide a qualitative examination of specific differences between groups.

Steps for ANCOVA

- *Consider the following:*

- All variables reliable*?*
- Are groups sufficiently homogeneous, i.e., low within group variance?
- Low to No correlation between IV & covariates?
- Notable correlation between DV & covariates?
- Can the design support causal inference (e.g., random assignment to manipulated IV, control confounds)?
- Are means significantly different, i.e., high between group variance?
  - Do groups differ after controlling for covariate?

ANCOVA Steps

- *Descriptive Statistics*
  - Means, standard deviations, skewness and kurtosis
- *Correlations*
  - If possible, across time for test-retest reliability
  - Across variables to assess appropriateness of ANCOVA
- *Test of Homogeneity of Regression*
- *ANOVA* (conduct as a comparison)
  - The ANOVA model with no covariate is a subset of the ANCOVA, using a measure of model fit such as adjusted R^2 or BIC could tell you whether adding a covariate to the model results in a a better one
- *ANCOVA* (ANOVA, controlling for covariates)
  - Conduct follow-up tests between groups
- Note effect size

ANCOVA Example

- This example will show the nature of ANCOVA as regression i.e. adhering to the general linear model
- ANCOVA is essentially performing a regression and, after seeing the difference in output between the stages in a hierarchical regression, we can better see what ANCOVA is doing

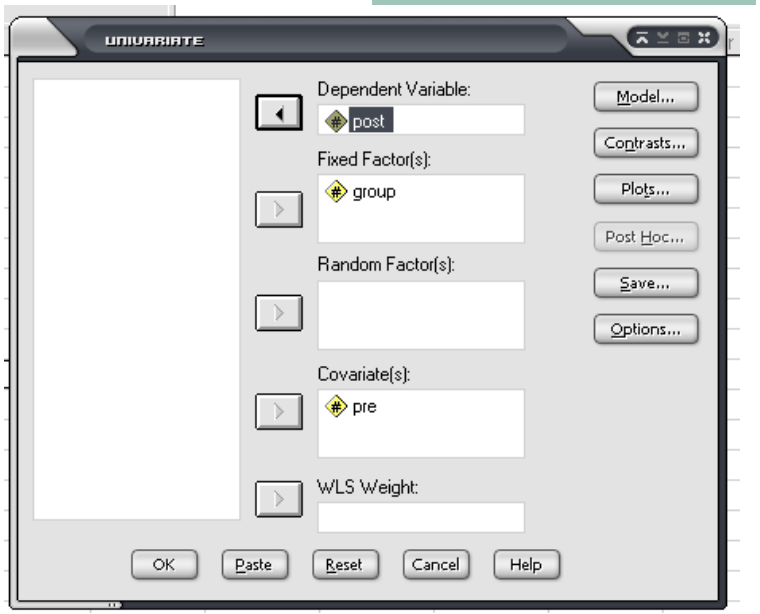
ANCOVA Example

- Consider this simple pretest-treatment-posttest setup
- We want to control for pretest scores at pre to see if there is a posttest difference b/t treatment and control groups

	Pre	Post
Treatment	20	70
Treatment	10	50
Treatment	60	90
Treatment	20	60
Treatment	10	50
Control	50	20
Control	10	10
Control	40	30
Control	20	50
Control	10	10

ANCOVA Example

- Dummy code grouping variable
  - Though given the two group set up, it probably should already be coded as such anyway for convenience



Tests of Between-Subjects Effects

Dependent Variable: post

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	4842.105[a]	2	2421.053	12.123	.005
Intercept	3112.762	1	3112.762	15.587	.006
pre	842.105	1	842.105	4.217	.079
group	4199.482	1	4199.482	21.029	.003
Error	1397.895	7	199.699		
Total	25600.000	10			
Corrected Total	6240.000	9			

a. R Squared = .776 (Adjusted R Squared = .712)

ANCOVA Example

- To get the comparison of the variance output from regression, we'll go about the MR in sequential fashion
- First enter the covariate, then run a regression with the grouping variable added to the model

ANCOVA Example

- Ancova output from GLM/Univariate in SPSS
- Regression output
- 4842.105-642.623 =4199.482

Tests of Between-Subjects Effects

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ANOVA[c]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	642.623	1	642.623	.918	.366[a]
	Residual	5597.377	8	699.672		
	Total	6240.000	9			
2	Regression	4842.105	2	2421.053	12.123	.005[b]
	Residual	1397.895	7	199.699		
	Total	6240.000	9			

a. Predictors: (Constant), pre

b. Predictors: (Constant), pre, group

c. Dependent Variable: post

ANCOVA Example

- Compared to standard regression output
- Note how partial eta-squared is just the squared partial correlations from regression
- The coefficient for the dummy-coded variable is the difference between marginal means in the Ancova

Tests of Between-Subjects Effects

Dependent Variable: post

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	4842.105[a]	2	2421.053	12.123	.005	.776
Intercept	3112.762	1	3112.762	15.587	.006	.690
pre	842.105	1	842.105	4.217	.079	.376
group	4199.482	1	4199.482	21.029	.003	.750
Error	1397.895	7	199.699			
Total	25600.000	10				
Corrected Total	6240.000	9				

a. R Squared = .776 (Adjusted R Squared = .712)

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients			Correlations	
		B	Std. Error	Beta	t	Sig.	Zero-order	Partial
1	(Constant)	10.316	9.184		1.123	.298		
	pre	.526	.256	.368	2.054	.079	.321	.613
	group	41.053	8.952	.822	4.586	.003	.801	.866

a. Dependent Variable: post

group

Dependent Variable: post

group	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
.00	23.474[a]	6.325	8.517	38.430
1.00	64.526[a]	6.325	49.570	79.483

a. Covariates appearing in the model are evaluated at the following values: pre = 25.0000.

ANCOVA

- So again, with ANCOVA we are simply controlling for (taking out, partialing, holding constant) the effects due to the covariate and seeing if there are differences in our groups
- The only difference between it and regular MR is the language used to describe the results (mean differences vs. coefficients etc.), some different options for analysis because of the categorical variable (post hocs, *d* effect size), and an additional assumption (homogeneity of regression)
- ANCOVA is primarily concerned with reducing error variance/increasing statistical power. When used appropriately for experimental setups, it will do this, however there is essentially no adjustment of means because, due to random assignment, the groups are equal on the covariate
  - In other words your effect size *d* is still the same

ANCOVA issues

- Using ANCOVA with intact groups is very problematic
  - You will have some non-zero correlation between the covariate and grouping variable
  - No reason to believe groups are similar on the covariates.
  - As such, you will be partialling out the treatment variance also
  - Also, the means will be adjusted differently, leading to interpretability problems
- Paradoxically, when it's needed least it is most appropriately used, and when desired most, it may lead to inaccurate assessment of treatment effects
- "One may well wonder exactly what it means to ask what the data would be like if they were not what they were"
  - N. H. Anderson 1963

- Gist: ANCOVA, like everything else, is not to be chosen as an analysis on a whim, or because it 'sounds cool', the model must make sense, the data appropriate, the conclusions viable.
- Things to consider
  - Why not just run the MR?
  - If experimental: the adjusted mean difference will be the same
  - If not: you likely violate the HoR assumption
  - Why not focus on the model change across groups instead?
    - Note changes of an MR model across the groups as in Multilevel Modeling